EC 3210 Solutions

Assignment 6

5.1 Suppose a resonator is made up of two mirrors with radii of curvature equal to 0.5 m and 1 m respectively. Calculate the range of mirror separations L that will provide stable operation. (Include the separations that will provide marginal stability.)

The g parameters are

$$g_1 = 1 - \frac{L}{r_1} = 1 - \frac{L}{0.5} = 1 - 2L \tag{1a}$$

and

$$g_2 = 1 - \frac{L}{r_2} = 1 - \frac{L}{1.0} = 1 - L\,. \tag{1b}$$

The product $g_1 g_2$ is

$$g_1 g_2 = (1 - 2L)(1 - L)\,. \tag{1c}$$

Looking at the plot of $g_1 g_2$ vs. L in Fig. 1, we can identify the regions where $g_1 g_2$ is between 0 and 1. We see that we want

$$0 \leq L \leq 0.5 \tag{2a}$$

and

$$1 \leq L \leq 1.5\,. \tag{2b}$$

There are *two* separated regions of mirror spacing that will provide stable operation.

5.2 Considering the rectangular and circular mode patterns shown in the text, ...

a. ... sketch the patterns expected for the TEM_{22}, TEM_{14}, and the TEM_{23} rectangular modes.

b. Repeat part (a) for circular modes.

a. For rectangular geometry, we have one more in the x-direction and y-direction than the index value. See Fig. 2.

b. For a TEM_{mn} circular mode we will have $m + 1$ peaks (with one on the center point) along $2n$ radial lines, as seen in Fig. 3. Note: I have added the TEM_{24} as well.

6.1. Calculate the Doppler-broadened linewidth of the HeNe 1.15 μm transition. Assume that the gas discharge equilibrium temperature is approximately 350K.

We want to calculate the Doppler-broadened linewidth of the HeNe 1.15 μm transition for $T = 350$ K.

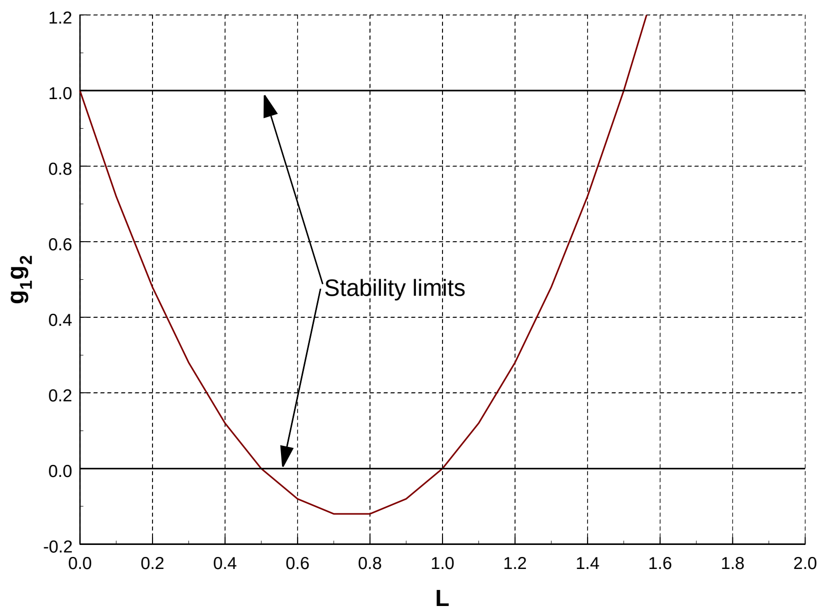

Figure 1: Problem 5.1. Plot of $g_1 g_2$ vs. L.

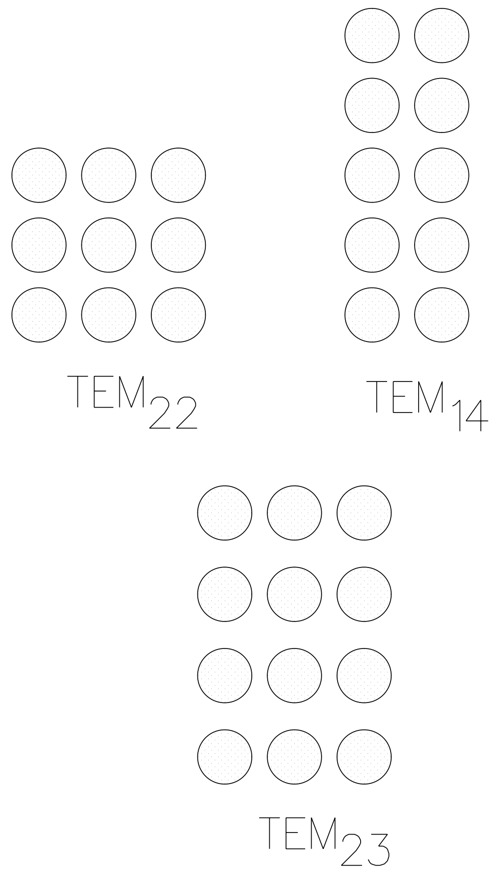

Figure 2: Rectangular mode patterns for Problem 5.2a.

We begin by calculating the center frequency.

$$\nu_0 = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{1.15 \times 10^{-6}} = 2.61 \times 10^{14} \text{ Hz}\,. \tag{3}$$

Now we need to find kT. We know that $kT = 1/40$ eV for $T = 300$K, so, at $T = 350K$, we have

$$kT_{350} = (kT)_{300}\left(\frac{350}{300}\right) = \left(\frac{1}{40}\right)\left(\frac{350}{300}\right) = 2.92 \times 10^{-2} \text{ eV}\,. \tag{4}$$

Next, we need to find Mc^2 for neon. (Neon is the lasing species in the helium and neon mixture. The helium is in the mixture only to pump the neon by a collision process; it does not lase.)

$$M(\text{neon}) = 20 \tag{5a}$$

$$Mc^2 = (20)(1 \times 10^9) = 20 \times 10^9 \text{ eV}\,. \tag{5b}$$

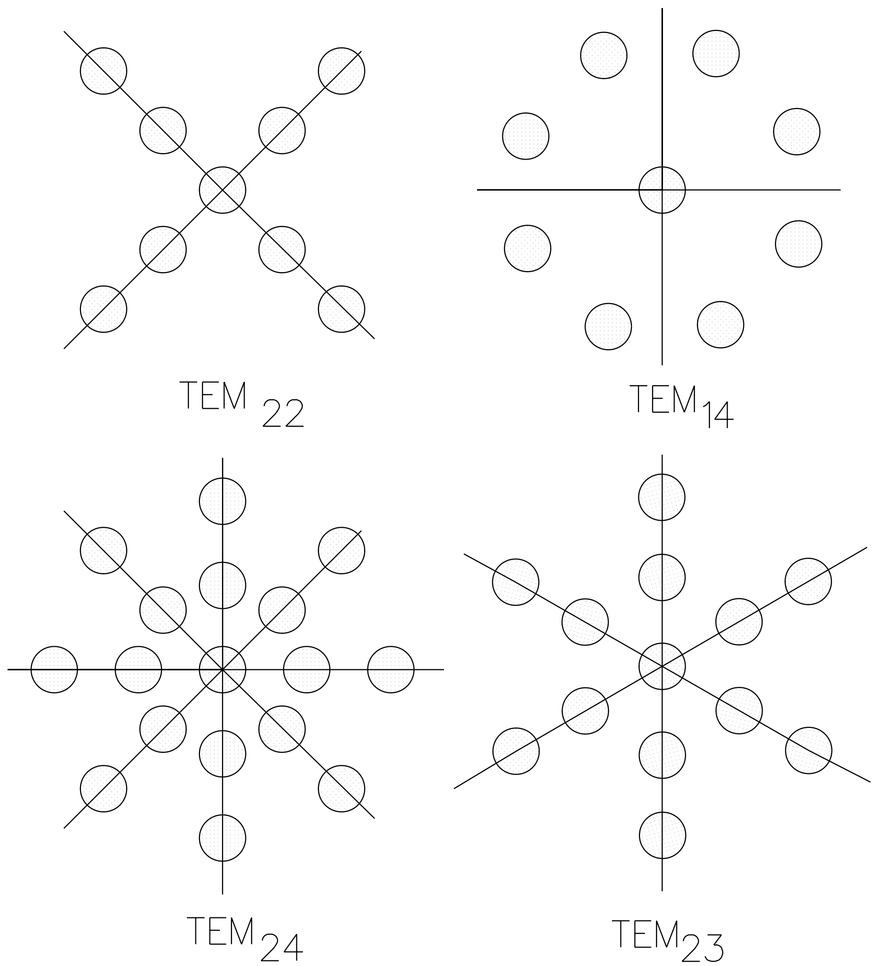

Figure 3: Circular mode patterns for Problem 5.2b.

Hence, we have

$$\frac{kT}{Mc^2} = \frac{2.92 \times 10^{-2}}{20 \times 10^9} = 1.458 \times 10^{-12}\,. \tag{5c}$$

The frequency linewidth $\Delta\nu$ is then found from

$$\Delta\nu_{\mathrm{Doppler}} = \nu_0\sqrt{\frac{8(\ln 2)(kT)}{Mc^2}} = (2.61 \times 10^{14})\sqrt{8(\ln 2)(1.458 \times 10^{-12})} = 7.42 \times 10^8 = 74.2\ \mathrm{GHz}\,. \tag{5d}$$

6.2. Consider a three-level ruby laser ($\lambda = 694$ nm). The lifetimes of the upper and middle levels are 50 ns and 3 ms, respectively.

a. What is the lifetime of the lowest level?

b. Calculate the frequency linewidth $\Delta\nu$ of the lifetime-broadened lineshape, $g(\nu)$.

c. Using a computer, plot $g(\nu)$.

We want to find the lifetime broadened Lorentzian curve for a three-level ruby laser ($\lambda = 694 \times 10^{-9}$ m) with $\tau_2 = 50$ ns and $\tau_1 = 3$ ms.

a. The lifetime of the lowest energy level (the ground state) is infinity;

$$\tau_0 = \infty \,. \tag{6a}$$

b. The linewidth is

$$\Delta\nu = \frac{1}{\pi}\left(\frac{1}{\tau_1} + \frac{1}{\tau_0}\right) = \frac{1}{\pi}\left(\frac{1}{3 \times 10^{-3}} + \frac{1}{\infty}\right) = 106.1 \text{ Hz} \,. \tag{6b}$$

c. The center frequency of the line is

$$\nu_0 = \frac{c}{\lambda} = \frac{3 \times 10^8}{694 \times 10^{-9}} = 4,32 \times 10^{14} \,. \tag{6c}$$

The graph of the Lorentzian lineshape is shown in Fig. 4.

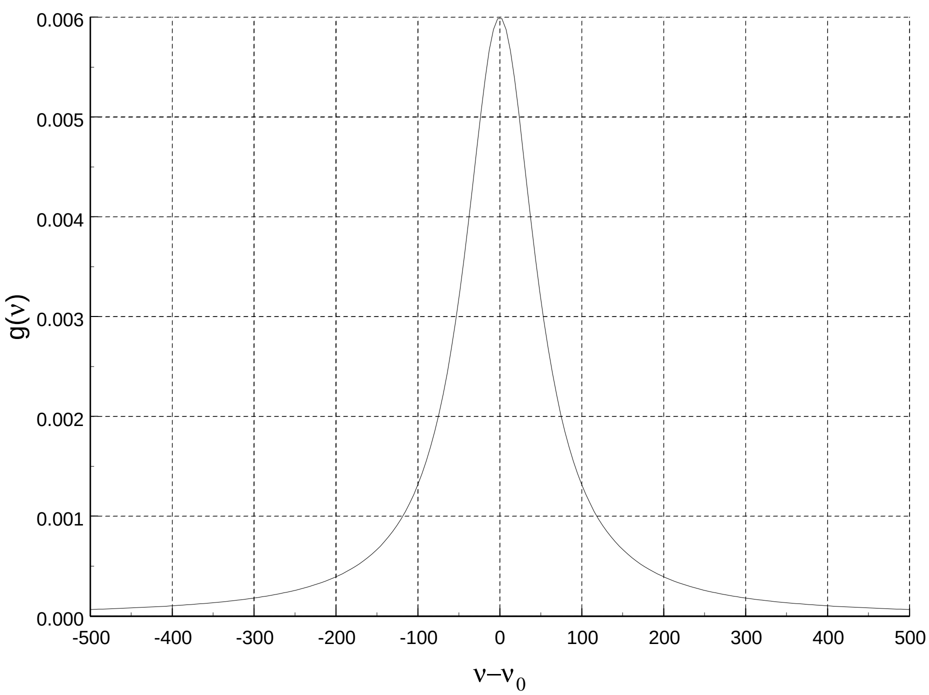

Figure 4: Lorentzian lineshape ($\nu_0 = 4.32 \times 10^{14}$ Hz) for Problem 6.2.